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*Published in:*

Interplay between Geotechnical Engineering and Engineering Geology : Proceedings of the 11th European Conference on Soil Mechanics and Foundation Engineering, Copenhagen, 28 May - 1 June 1995 : XI ECSMFE

*Publication date:*  
1995

*Document Version*  
Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*

Ibsen, L. B. (1995). The Static and Dynamic Strength of Sand. In *Interplay between Geotechnical Engineering and Engineering Geology : Proceedings of the 11th European Conference on Soil Mechanics and Foundation Engineering, Copenhagen, 28 May - 1 June 1995 : XI ECSMFE : DGF-bulletin 11* (pp. 69-76). Dansk Geoteknisk Forening. DGF-bulletin Vol. 6 No. 11

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# The Static and Dynamic Strength of Sand

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**SYNOPSIS:** This paper describes a number of new characteristic phenomena of sand exposed to static and dynamic loadings. The characteristic phenomena have been discovered by triaxial cell testing at the Soil Mechanics Laboratory, Aalborg University using test specimens with equal height and diameter. The tests, CD,  $CU_{u=0}$  and dynamic CU tests are run with a constant deformation speed varying from 4 to 100.000% ph. By studying the basic phenomena in triaxial tests under uniform conditions identical responses of the soil due to static and dynamic loadings have been discovered. The static and dynamic responses of sand can be explained by the characteristic state and the strength of sand under undrained conditions is found to be controlled by the drained failure condition both for static and dynamic loadings.

## 1. INTRODUCTION

During the last 20 years the use of computer programs to solve geotechnical problems has increased tremendously. This development results in strong needs for reliable soil models which describe the basic phenomena related to soil behaviour during static, cyclic and dynamic loadings.

Laboratory testing makes it possible to study soil behaviour in detail under control and well known conditions. By studying the basic phenomena in triaxial tests under uniform conditions identical responses of the soil due to static and dynamic loadings have been discovered. The static and dynamic responses of sand will be explained by the characteristic state, and it will be explained why the strength of sand under undrained conditions is controlled by the drained failure condition both for static and dynamic loadings. The consequence of this study is that the design practice used today for foundations subjected to variable or dynamic loads is too conservative.

## 2. TEST PROGRAM

The tests performed and described in this paper are normal static CD tests,  $CU_{u=0}$  and dynamic CU triaxial tests. The dynamic CU tests are performed as undrained tests where the pore pressure is measured, whereas the  $CU_{u=0}$  tests are carried out by measuring the volume change and controlling the cell pressure in such a way that  $\Delta\epsilon_v = 0$  throughout the test. In this way the undrained condition is ensured and the effective stress path is measured.

In triaxial tests - in which the loads, movements, volume- and pore pressure changes are measured on the outside of the specimen - it is obvious that homogeneous conditions must exist inside the specimen throughout the test in order to calculate the correct values of the stresses, strains and void ratio. Although it has been emphasized for at least 20 years that triaxial tests must be performed on specimens with a height equal to the diameter and smooth pressure heads, Jacobsen (1970), Jacobsen (1981) and Lade (1982), it

is generally considered that adequately uniform conditions will be achieved by using tall specimens with heights greater than or equal to two diameters.

Ibsen (1994) found that specimens with double height developed nonuniformities in strains resulting in measurements of non correct stress/strain relations of the material. During undrained conditions the nonuniform development in strains results in inner draining and the correct generation in the pore pressure cannot be produced. The consequence of the errors is that studies of basic phenomena in soil, drained or undrained, must be performed on specimens with a height equal to the diameter and with smooth pressure heads. This is the only way to ensure that soil modeling does not reflect test errors but the real properties of the soil.

To ensure homogeneous stress and strain conditions the tests are performed on cylindrical specimens with a height of 70mm and a diameter of 70mm, and bounded by lubricated caps and bases. The tests are run in an automatic, newly developed version of the Danish Triaxial Apparatus, Ibsen (1994).

The difference between the static and the dynamic device is that the dynamic triaxial apparatus is loaded by hydraulics. The static tests are run by a constant deformation speed at 4% ph while the dynamic tests are run with a constant speed varying from 40 - 100,000% ph. The measuring systems in the two test apparatuses are identical and carried out by transducers. The triaxial cell is constructed by M. Jacobsen in agreement with the principles described in Jacobsen (1970).

The study described in this paper is based on tests run with two uniform sands called Baskarp No 15 and Lund No 0. The index properties are shown in the Table 1.

The test specimens were prepared by a pluvial technique and carefully saturated in vacuum. This technique ensures homogeneous and totally saturated specimens.

Table 1. Index Properties.

	Baskarp No 15	Lund No 0
$d_{50}$ mm	0.140	0.400
$C_U$	1.780	1.700
$d_s$	2.650	2.650
$e_{max}$	0.860	0.820
$e_{min}$	0.549	0.550

The triaxial test series were performed on isotropically consolidated samples with the relative densities shown in Table 2. The test results are reported in Ibsen (1990), Ibsen and Bødker (1994), Jakobsen and Simonsen (1994).

Table 2. The static triaxial test series are performed with the following relative densities.

	Relative densities
Baskarp No 15	0.01, 0.51, 0.8
Lund No 0	0.48, 0.78, 0.93, 1.0

The parameters, which describe the state of the soil under axisymmetrical stress conditions, are calculated from the measurements taken during the tests. These parameters are:

the cell pressure  $\sigma'_3$

the mean normal stress

$$p' = 1/3(\sigma'_1 + 2\sigma'_3)$$

the deviator stress  $q' = \sigma'_1 - \sigma'_3$

the volumetric strain  $\epsilon_v = \epsilon_1 + 2\epsilon_3$

the deviator strain  $\epsilon_q = 2/3(\epsilon_1 - \epsilon_3)$

where  $\sigma'_1$  is the vertical and  $\sigma'_3$  the horizontal principal stresses. The stresses are effective and  $\epsilon_1$  and  $\epsilon_3$  are the principal strains.



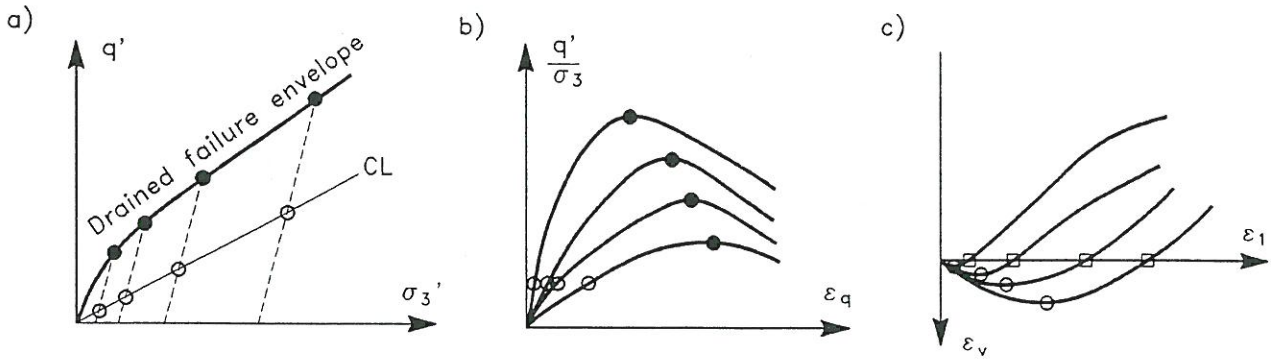


Fig. 1. The figure outlines results of four drained triaxial tests. The tests illustrate the development of CD-tests carried out on dense sand performed with different levels of confining pressure, and on specimens with equal height and diameter.

### 3. THE CHARACTERISTIC STATE

Fig. 1 shows the results of four drained triaxial CD-tests. The tests are performed with different levels of cell pressure  $\sigma_3'$  which are held constant throughout the test. In each test failure is seen to be well defined as the state in which the deviator stress  $q'$  is a maximum, see Fig. 1b. In the figure the performance curve is divided by  $\sigma_3'$  which causes the curve with the smallest confining pressure to be situated highest. The strain at failure is seen to be a function of  $\sigma_3'$ . The values  $p'_f, q'_f$  at failure describe a curved failure envelope, which defines the stress space, limiting the stress combination  $p', q'$ , see Fig. 1a.

Laboratory tests on several sands have shown that a characteristic threshold exists in granular materials which is defined as the stress state where the volume change goes from contraction to dilation. On the curve  $\epsilon_1 - \epsilon_v$  curve, Fig. 1c, the characteristic threshold is marked with open circles at the point in which the sample has minimum volume. The stress stage  $p'_c, q'_c$  where  $\delta\epsilon_v = 0$  is defined and described as the *Characteristic State*, Luong (1980).

If the characteristic states are plotted in the stress space they describe a straight line through 0,0, see Fig. 1a. The line is described as the *Characteristic Line CL*, Luong (1980) and divides the stress space into two

subspaces in which the stress combinations lead to different deformation mechanisms.

Under the characteristic line the stress combinations lead to contraction and  $\delta\epsilon_v > 0$ .

Above the line the stress combinations lead to dilation and  $\delta\epsilon_v < 0$ .

Under the characteristic line the resistance to deformation is governed by sliding friction due to microscopic interlocking depending upon surface roughness of contracting particles or interlocking friction afforded by adjacent particles. The resistance is due to pure friction and the characteristic state describes an intrinsic parameter which defines a characteristic friction angle  $\phi_{cl}$  which determines the interlocking capacity of the grains. As  $\phi_{cl}$  is a token of pure friction the value of  $\phi_{cl}$  is independent of the initial sand density and the interlocking capacity is identical in triaxial compression and extension. These postulates have been verified by the four CD-test series executed and the observation is in agreement with Loung (1980). The interlocking capacity is found to be:

Baskarp No 15 :  $\phi_{cl} = 30.4^\circ$

Lund No 0 :  $\phi_{cl} = 29.5^\circ$

In the subspace situated between the failure envelope and the characteristic line the

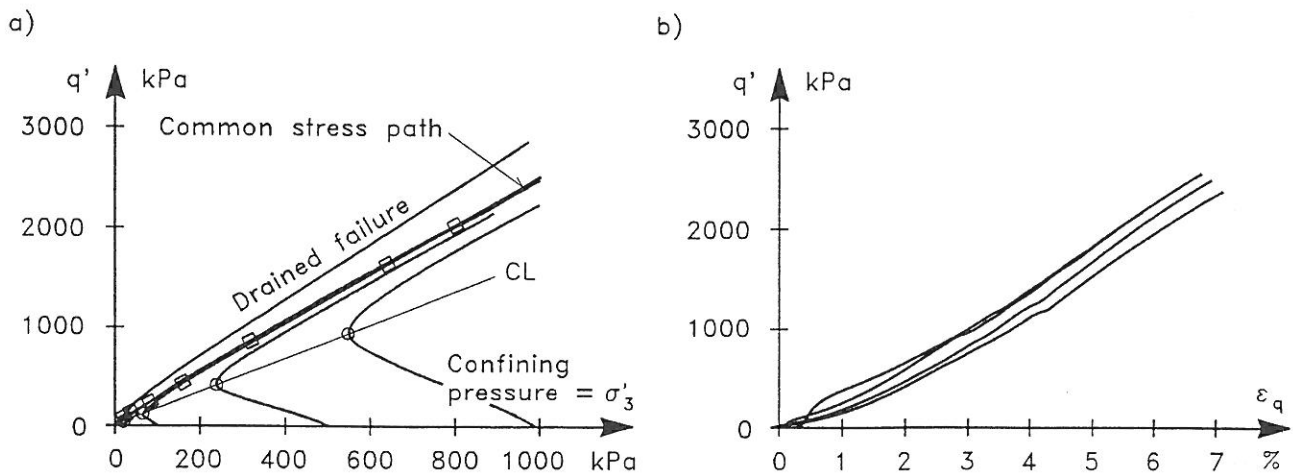


Fig. 2. Results of 4  $CU_{u=0}$  tests performed on Lund No 0 with  $I_D = 0.78$ . The tests were performed on specimens with equal height and diameter.

resistance to deformation is governed by interlocking disrapture where the individual particles are plucked from their interlocking seats and made to slide over the adjacent particles leading to large dilative volume changes. The resistance to the deformation and thereby the size of the subspace is strongly dependent on the initial sand density as it requires more energy for the grains in a dense sand to move the adjacent particles while sliding over each other than for the grains in a loose sand. This subspace does not exist in a very loose sand where no dilation occurs and in this case the failure envelope will be identical to the characteristic line.

#### 4. THE CHARACTERISTIC STATE UNDER UNDRAINED CONDITIONS

Under undrained conditions the characteristic state is also an intrinsic parameter which controls the effective stress path.

During a conventional undrained triaxial test the pore pressure increases at first in order to prevent the sand from contraction and  $\delta u > 0$ . When the deviator stress approaches the characteristic line  $\delta u \rightarrow 0$ . In Fig. 2, where the results of four undrained triaxial  $CU_{u=0}$ -tests are shown it appears that the stress stage, where  $\delta u = 0$ , is identical with the interlocking capacity in the sand,

see Fig. 2.a. If  $q'$  increases further the effective stress path is situated in the subspace which is characterized by dilation. In order to prevent interlocking disrapture the pore pressure generation becomes  $\delta u < 0$ , see Fig. 2.a.

#### 5. THE COMMON STRESS PATH

Fig. 2a shows that the effective stress path approaches a common stress path asymptotically, defined by the stress stage marked with open squares. These stress stages are defined by the drained strain characteristic where  $\sum \delta \epsilon_v = 0$ , see Fig. 1c. This common stress path is normally considered to be the undrained failure envelope. In Fig. 2b it is shown that the common stress path does not represent any failure state in the sand, and tests covering a  $\sigma'_3$  interval from 5 kPa to 2000 kPa do not reveal any maximum at the performance curves. This conclusion is not in agreement with the state of the art. The state of the art is build on undrained tests run on specimens with double height and thereby it reflects the test errors and not the real properties of the soil, see Ibsen (1994).

#### 6. FAILURE UNDER UNDRAINED CONDITIONS

Failure under undrained conditions develops

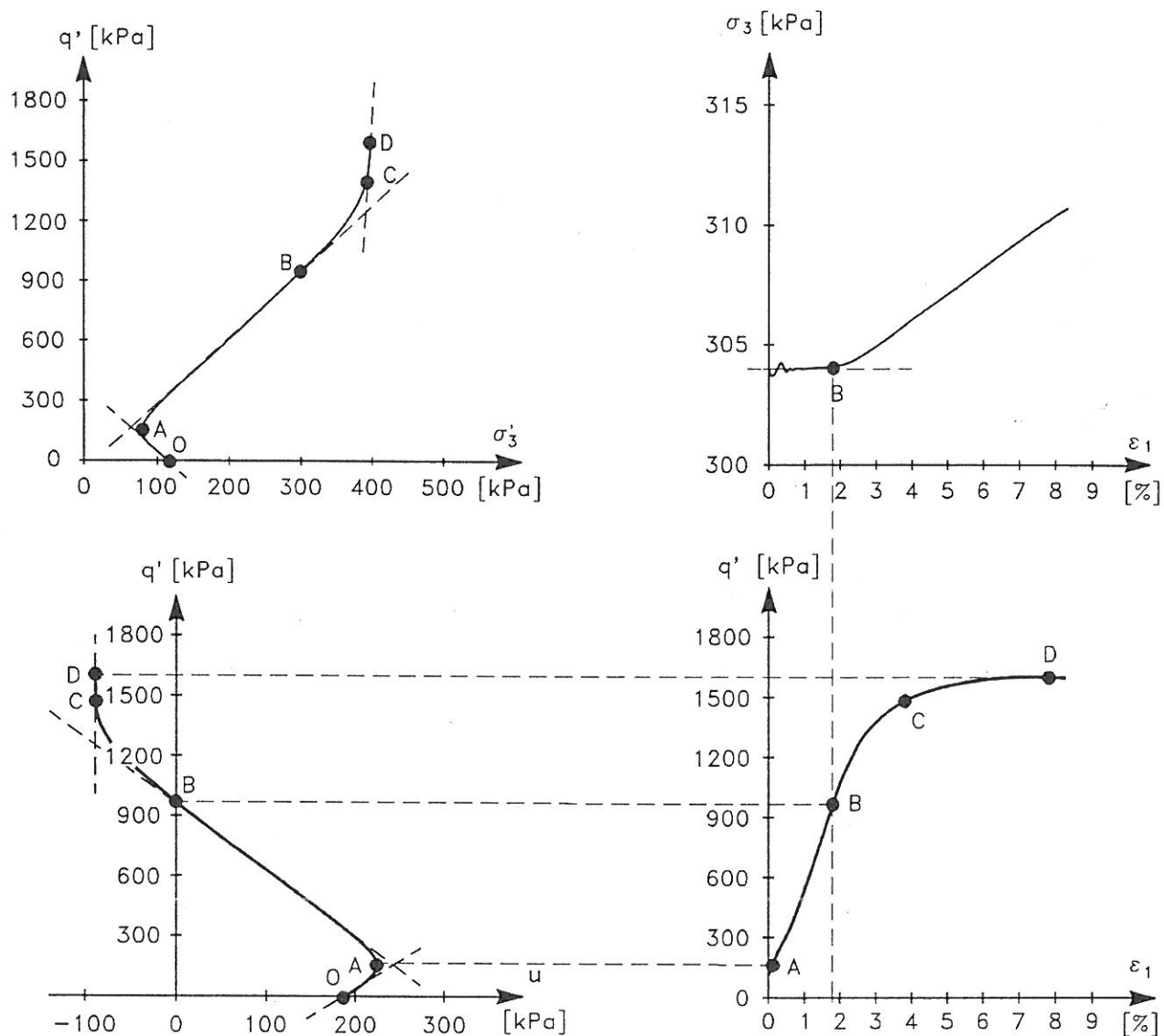


Fig. 3. Results of a dynamic CU triaxial test performed on Baskarp No 15. The tests were run by a constant deformation speed at 1000% ph. The specimen was prepared with  $I_D = 0.8$ .

when the pore water fails to prevent the interlocking disruption. From this state  $\Delta\epsilon_v$  is no longer equal to zero and the soil starts to dilate. In this way the failure condition is controlled by the same mechanism as failure under drained conditions. This development can be studied in Fig. 3.

The variation of the pore pressure during a dynamic CU test is outlined in Fig. 3. The figure shows an initial increase of the pore pressure in order to prevent the sand from contraction and  $\delta u > 0$ . When the deviator stress approaches the characteristic line, point A,  $\delta u \rightarrow 0$ . Between point A and B

the pore water manages to prevent interlocking disruption and the pore pressure generation is  $\delta u < 0$ . Up to point B the stress-strain curve, Fig. 3d, develops the same strain hardening response as outlined in Fig. 2b, and the stress path follows the common stress path. When the pore pressure becomes negative the response changes character. The pore water can no longer prevent interlocking disruption and the material starts to dilate slightly. The dilation causes a strengthening of the material and the response starts to deviate from the common stress path. At point C the pore water performs the max-

imum resistance - 90 kPa. From this state the strengthening of the material depends entirely on the dilation. At point D failure occurred. The negative pore pressure does not vanish during failure which is a point of special interest.

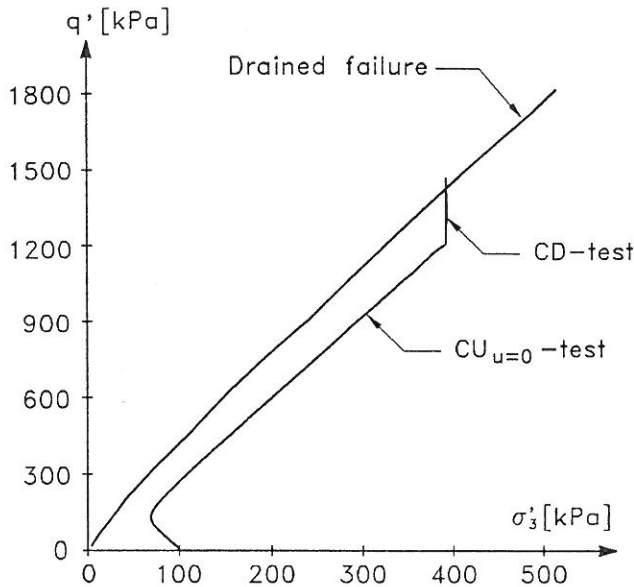


Fig. 4. Static  $CU_{u=0}/CD$  test run by a constant deformation speed at 4% ph. The specimen is prepared with  $I_D = 0.8$ .

The state of the art opinion is as follows: If  $u \rightarrow -100$  kPa then the pore water will transform into gas. The soil will generate a cavitation and the negative pore pressures will disappear. As outlined in Fig. 3 this mechanism does not develop. As seen the pore water performs a constant resistance of -90 kPa during failure, see point D.

The  $CU_{u=0}/CD$  test outlined in Fig. 4 supports the mechanism described above which controls failure under undrained conditions. It is seen that the shift from undrained to drained conditions results in dilation and the observed response is identical to what is outlined in Fig. 3.

## 7. DYNAMIC RATE EFFECTS

The dynamic rate effect has been studied by performing triaxial tests with a constant deformation speed varying from 40 to 100.000%

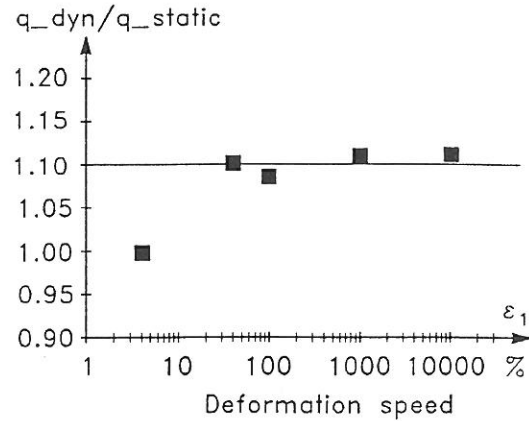


Fig. 5. The failure values of dynamic  $CU$  triaxial tests run with a constant deformation speed varying from 40 to 100.000% ph.

ph. The result of this study is outlined in Figs. 5 and 6. No rate effect has been found on the strength of the sand. The ratio between the static and the dynamic strengths is 1.1 in an interval from 40 to 100.000% ph, see Fig. 5. The 10% increase of the dynamic strength is a result from the fact that the sand does not have the same void ratio at failure. A similar increase of strength is shown in the  $CU_{u=0}/CD$  test, see Fig. 4.

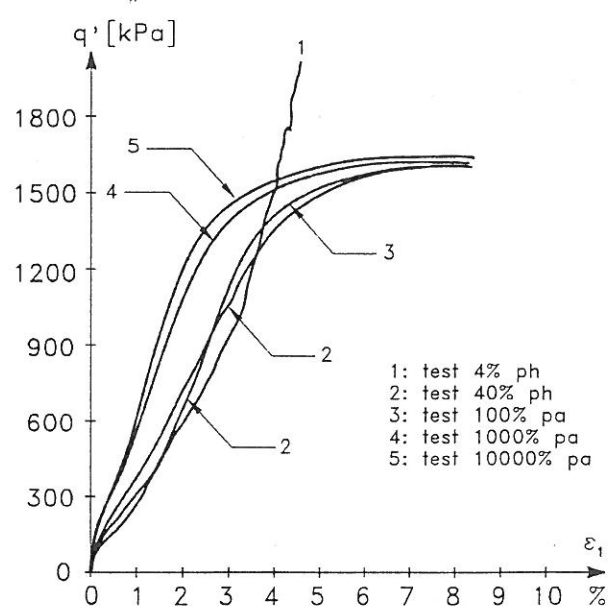


Fig. 6. Stress-strain curve of the dynamic triaxial test outlined in Fig. 5.

On the other hand the tests revealed that the stress-strain curve shows a great rate effect,



see Fig. 6. This effect is important in analyzing the interaction between the structure and the soil when calculating the dynamic amplification factors.

## 8. CONCLUSIONS

By studying the basic phenomena in triaxial tests under uniform conditions identical responses of the soil due to static and dynamic loadings have been discovered. The static and dynamic responses of sand can be explained by the characteristic state, and the strength of sand under undrained conditions is found to be controlled by the drained failure condition for both static and dynamic loadings.

The negative pore pressure does not vanish during failure as stated by the state of the art which is of special interest. This means that the negative pore pressures are present during failure. Thus, the negative pore pressure can be taken into calculation in the design of foundations subjected to variable or dynamic loads as a reliable stabilizing force.

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On the other hand the tests revealed that the stress-strain curve shows a great rate effect. This effect is important in analyzing the interaction between the structure and the soil when calculating the dynamic amplification factors.

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